Effect of Drag-Reducing Additives on Boundary-Layer Turbulence

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A preliminary investigation has been made concerning the effects of polyethylene oxide on the spectral density of turbulent fluctuations in a boundary layer. The Polyox solution was injected into a two-dimensional boundary layer along a flat plate which was towed at 9.5 fps in the Naval Academy's 85-ft tow tank. Measurements of the mean and fluctuating components of the velocity near the wall were made with conical hot film sensors. Polymer injection resulted in an increase in rms level of the fluctuations and in an increase in spectral density below 200 Hz. Above this frequency, the spectral density decreased, the effect being more pronounced at higher frequencies and at higher polymer concentrations. Measurements of the mean and fluctuating components of the wall shear stress were made with flush-mounted hot-film sensors submerged in the viscous sublayer. Polymer injection resulted in a decrease in mean shear, in rms level, and in spectral density of the fluctuating wall shear stress. The latter effect increased with increasing frequency and all effects increased with higher molecular weight of the additives. All results suggest the polymer shifts the scale of turbulence away from the energy-dissipating small eddies.

Nomenclature

A = bridge voltage squared at zero flow
B = King's formula constant
db = decibels
e = fluctuating voltage
e_L = fluctuating linearized voltage
E_b = anemometer bridge voltage
$\bar{E}_b$ = time-averaged bridge voltage
E_L = linearized voltage
f = frequency
$F_e(f)$ = normalized voltage spectral density
$F_u(f)$ = normalized velocity spectral density
$G_x(f)$ = power spectral density of quantity x in a 1-Hz band
$G_x(f)\Delta f$ = power spectral density of quantity x in a $\frac{1}{10}$-octave band
k_1 = one-dimensional wave number
K = constant
m = linearizer exponent
n = King's formula exponent
Δt = temperature difference between sensor and environment
$T.I.$ = turbulence intensity
u = fluctuating local velocity
U = local velocity
$\bar{U}$ = time-average local velocity
U_c = convection velocity
τ_w = wall shear stress
$\bar{\tau}_w$ = time-averaged wall shear stress
τ_w' = fluctuating wall shear stress

Introduction

THE so-called "Toms Effect" has received considerable attention in recent years, especially as a means of reducing drag and possibly flow noise on naval vehicles. The fifty-odd references included in this paper demonstrate a variety of experiments, theories, and speculations concerning the enigmatic influence of polymer additives on turbulence. Although most naval applications involve the injection of the drag-reducing additive into a boundary layer, nearly all the experimental investigations to date have been gross measurements using essentially homogeneous solutions in pipe flows.[1-15] Shin[16] studied drag reduction in Couette flow, and Doherty[17] and Vogel and Patterson[18] have reported on the drag reduction attained through the injection of polymer additives into the boundary layer of streamlined bodies moving at relatively low Reynolds numbers. The drag reduction and pitot traverse experiments have given rise to three generally accepted conclusions: 1) maximum drag reduction increases with increasing Reynolds number and molecular weight of any one class of polymers,[3,10,16] 2) there is a critical wall shear stress below which drag reduction does not take place,[1,2,4,14,19] and 3) the viscous sublayer increases in thickness with addition of the additive.[1,2,12,15,20]

These gross observations have given rise to several speculations concerning the nature and cause of the phenomenon of drag reduction. Early investigators[12,13] suggested an effective slip at the wall, but this has been discounted.[21] The possibility of an anisotropic viscosity[16] and delayed transition[4,7,11,12,22] have also been explored but found inadequate to explain most of the experimental results. The most popular theory to date has been to attribute drag reduction to the viscoelastic properties of the polymer solution. This is not a simple explanation, however, and the theory takes many forms. A constitutive equation for the polymer influence has not yet been agreed upon, but viscoelasticity is considered to affect the viscous sublayer,[1,2,12,15,20,23-25] the flow relaxation time,[2,4,5,7,19,23] the turbulent energy spectra,[4,7,14,18,20,23,26-29] and the production and dissipation of turbulent energy.[4,7,14,21,23,24,30,31] A dual explanation considering both inertial[23] and viscoelastic[28] effects of the polymer additive on vortex stretching in the high shear region of the boundary layer will be discussed in this paper.

The popularity of the viscoelastic theory has not been matched by the detailed measurements required to establish

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Presented as Paper 67-459 at the AIAA 3rd Propulsion Joint Specialist Conference, Washington, D.C., July 17-21, 1967; submitted August 7, 1967; revision received March 7, 1968. This research was supported by the Office of Naval Research and the Naval Ship Systems Command.

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the influence of local polymer concentration on turbulent scales and intensity. It has been very difficult until recently to make meaningful turbulence measurements in liquids. The development of the constant temperature anemometer and the quartz-coated hot-film sensors which prevent electrolysis in conducting fluids has made possible investigations into the mechanism of drag reduction. The earliest of the hot-film studies was made by Vogel and Patterson[18] in the wake of a streamlined body with and without injection. The use of the conical hot-film sensor to eliminate lint catching and vortex shedding problems was pioneered by Fabula[20] in his study of decaying wakes. Turbulence measurements in pipes have been made by Patterson and Zakin[8,9] and Virk.[14]

To date, however, no turbulence investigations dealing with the effect of injecting polymer additives into a developing boundary layer have been reported in the literature. This is the subject of a series of experiments at the Naval Academy[23,26,27,32,33] and this is a report of the results of the first in the series.

In this investigation, the conical hot film suggested by Fabula[20] was used to traverse the boundary layer, and a flush-mounted hot film was used to measure the influence of the additives on the fluctuating wall shear stress. The use of flush-mounted heated elements was pioneered by Ludwieg,[34] Myers, Schauer, and Eustis,[35] Liepmann and Skinner,[36] Bellhouse and Schultz,[37] and Brown[38] in studies in air. The flush-mounted hot film has been used in water to detect the presence of laminar and turbulent flow by Townsin,[39] Burns and Murphy,[40] and Runstadler, Kline, and Reynolds.[41]

Experimental Procedures

A. Polyox

Polyethylene oxide, a product of Union Carbide Corporation, was chosen as the drag-reducing agent largely because of the positive results obtained by other investigators. Since the most significant effects were expected with the longest-chain polymer, polyox coagulant (molecular weight $\sim 7 \times 10^6$) was used initially. Some tests were made with WSR 301 (molecular weight $\backsim 4 \times 10^6$) for comparison.

Shin[16] made a study on the mechanical and biological degradation of polyox solutions, and concluded that both mechanisms had a pronounced effect on the polyox chains. Tests made using both aqueous formaldehyde and pure water as solvents showed that for the former there was little degradation due to aging (bacterial action) whereas biological degradation of the latter was continuous and pronounced. The solvent used in these studies, therefore, was a weak solution of formaldehyde in water and the solute was untreated polyox powder.

Although polyox is soluble in water, the particles must be well dispersed in the solvent to promote complete wetting. A lump of powder will only become wet on the outside and is thus impossible to dissolve easily. A large tank was used in order to obtain maximum surface area of solvent. Sufficient powder was sifted through a fine mesh screen to cover the surface with a thin layer. After standing for about $\frac{1}{2}$ hr a paddle was used to sink the surface layer. This process was continued until all the polyox had been wetted. The solution was gently stirred occasionally until a uniform viscosity was obtained.

Initially, the polyox coagulant was mixed in a 0.5% solution by weight, and injected at this concentration. Later, the solution was diluted to a 0.1% concentration to give better injection properties. The polyox WSR-301 was mixed and injected at a 0.1% solution by weight. No viscosity or molecular weight tests were run on the solutions.

B. Tow Tank

The test vehicle was limited in size and speed by the Naval Academy's tow tank in which all the tests were run. This tank is 85 ft long by 6 ft wide and 42 in. deep. The carriage is suspended on a single overhead rail by two sets of nylon wheels. A variable speed d.c. motor powered the two rubber drive wheels contacting a similar rail at the side of the tank (Figs. 1 and 4).

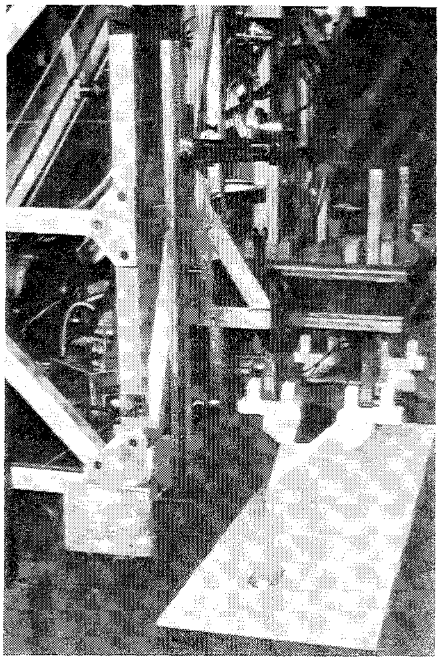

Fig. 1 Carriage and tank.

The velocity of the model was controlled to within 0.6% of the 5.5-knot value used for calculations. The velocity readout was obtained by passing a photocell beam through the slitted periphery of a rotating disk. This disk rotated with motion of the carriage and its 2000 evenly spaced slits cut the beam at a rate proportional to the speed of the carriage. The photocell pulse output was fed into a Hewlett-Packard model 5214L electronic counter and printout. The outputs of these two instruments gave a constant monitor of model velocity as it changed with polyox injection.

Due to the size of the two tank area, it was impossible to exercise any control over the environmental temperature. The water temperature during the testing period varied from 66° to 72°F. However, the effect on the additive caused by this temperature variation was apparently small and has been neglected.

C. Test Model and Subcarriage

The basic requirements for the model were for a large flat surface with a two-dimensional boundary layer and access to this surface for test probes. The final design consisted of a 1-in. plank, 4 ft long and 1 ft wide, topped by a streamlined hollow superstructure running the length of the model (Figs. 2 and 3). To insure a high degree of turbulence, a sawtoothed vortex generator was imbedded 5 in. downstream of the leading edge of the model.

Since previous work[17] with the present towing carriage had showed it to have marked vibration problems, it was felt that a subcarriage was needed. This subcarriage assembly was designed to eliminate as many of both the low- and high-frequency vibrations as possible. The high-frequency vibrations were reduced in amplitude through the use of two plates of $\frac{3}{4}$-in. plywood, with the bottom plate attached to the main carriage and the top one resting on this rigid plate supported by a 2-in.-thick block of foam rubber at each end. The longitudinal low-frequency vibrations of the model were nearly eliminated by suspending it on four freely swinging support arms. These support arms pass through slots in both planks and are pivoted at the top plank and at the sail.

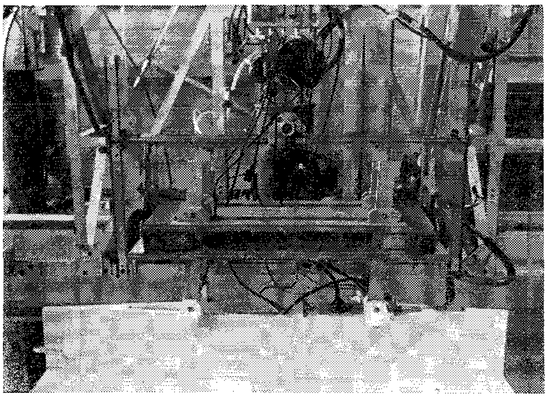

Fig. 2 Side view of carriage.

Tests showed this subcarriage to be very effective in minimizing carriage vibrations. The rms level of the vibrations as measured by the conical hot film in the freestream outside the boundary layer was only three times the electronic noise of the anemometer.

The injection system is illustrated in Fig. 4. A pressure tank supplied air through a standard regulator to a 2600-ml. graduated cylinder which was filled with the polyox solution. A remotely controlled solenoid valve on the outlet of the graduated cylinder gave accurate control of injection time. The solution then passed into a tank imbedded in the test model and into the boundary layer through a very fine wire mesh (Fig. 5). Injection rates were controlled by the air pressure regulator and calculated by timing the opening and closing of the solenoid and reading the total flow out of the graduated cylinder.

D. Instrumentation, Data Acquisition, and Analysis

For this project, flush-mounted and conical hot-film sensors, constructed by Thermo Systems Inc. to project requirements, were used as turbulence detectors. Three of the flush-mounted sensors were in the positions shown in Fig. 3 and aligned with their major axes perpendicular to the freestream flow. The conical sensor was used to traverse the boundary layer.

The signal was picked up from the hot film by a special 115-ft cable which traveled in a trough mounted over the tank and fed into a Thermo Systems Inc. model 1010 constant temperature anemometer. The signal was linearized and squared in the complementary model 1005B linearizer, amplified by a Columbia Research Laboratory model 6040 amplifier and recorded on an Ampex model FR-1100 tape recorder modified to use a 12-ft loop of No. 111 Scotch professional magnetic tape (Fig. 6).

The hot-film sensors were cold balanced frequently to check for drift, and an overheat of 10% was used in the bridge circuit. Before making a data run, the sensor was balanced at the speed and polyox injection rate to be used in the run. The initial runs of each day were made without injection and in these, the outputs of the amplifiers were adjusted to give a 1-v rms signal for each sensor. The tape recorder electronics had been previously adjusted to give unity gain. These initial amplifier settings were used in all subsequent runs in the series, thus referencing them to the 1-v rms level, established with no injection.

It may be noted here that no absolute calibration was made for the flush-mounted sensors. The exact relationship between the electrical output signal from the anemometer and the wall shear stress is still under study at the Naval Academy as another project. The conical sensors were calibrated to check the linearizer operation which proved satisfactory. A more complete calibration of conical and parabolic sensors has recently been completed by Dreyer.[32] It was assumed, based on work by Fabula,[20] that the dilute polyox solutions had a negligible effect on the hot-film response. Consequently, any frequency response irregularities should affect all runs equally, and relative response comparisons should be valid.

To account for the high-frequency decrease of power spectral density, the squared signal was also fed into a Krohn-Hite model 330NR band pass filter. A low cut-off of 250 cps and high cut-off of 10,000 cps were used after which the signal was again amplified another 20 db and recorded on a separate channel.

The tape loops were analyzed by a General Radio type 1911-A recording sound and vibration analyzer and were recorded directly in db vs frequency. Output signals were monitored on a Hewlett-Packard model 3400A-RMS voltmeter and a Tektronix type RM565 dual-beam oscilloscope.

Experimental Results

Figure 7 shows representative spectral densities of the electrical signal from the flush-mounted hot film with and without polymer injection. The spectral densities are based on a $\frac{1}{10}$-octave bandwidth as is explained in Appendix B. Unfortunately, it was not possible to convert the data to the spectral density of the fluctuating wall shear stress since the probes were uncalibrated. The equations for the transfer function between the anemometer bridge signal and the property being measured are given in Appendices A and B. An attempt to calibrate the flush-mounted sensors was made by Mies[33] but the calibration constants were found to vary considerably between sensors. This fact prevented recalculating the results of this experiment based on a later calibration.

Although the spectra without injection are nearly identical at both 8 in. and 16 in. from the boundary-layer trip, the effect of the polymer injection decreases as the flow moves aft on the model. This indicates that the polymer is most effective when it is concentrated in the viscous sublayer and wall turbulent regions where maximum production and dissipation of turbulence energy occurs. As the polymer diffuses outward in the downstream direction, the mean concentration approaches the actual concentration, and the influence of the polymer decreases. In all cases, the bridge voltage (which is proportional to some power of the local wall

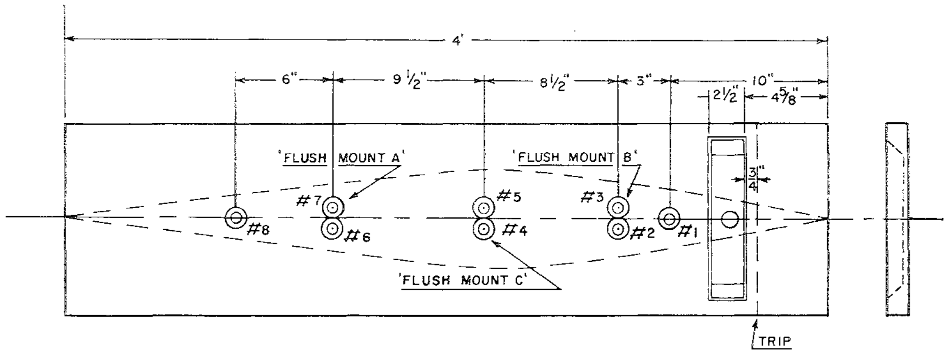

Fig. 3 Bottom detail of model.

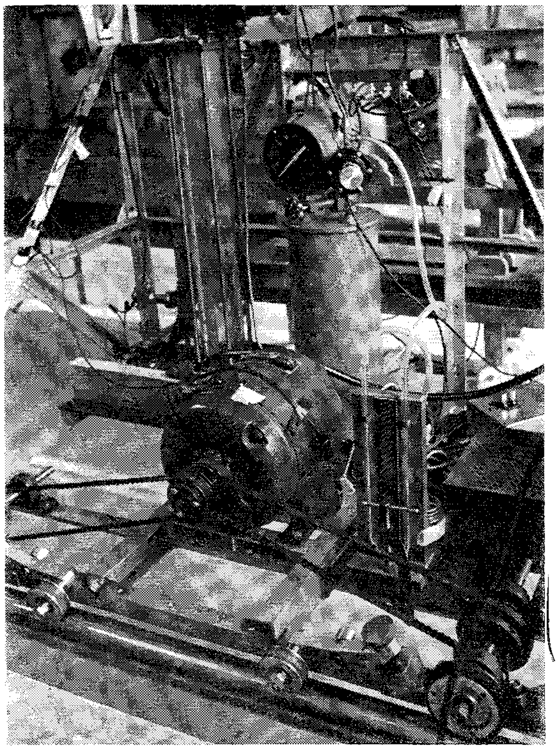

Fig. 4 Injection system.

shear stress) and the rms level of the fluctuating signal decreased when the polymer was injected.

Figure 8 shows that the ratio of spectral densities (without injection to with injection; perhaps the ratio should have been inverted) increases with increasing frequency. This indicates a reduction in the spectral energy content of the viscous sublayer fluctuations which increases with frequency (wave number). The shift away from high-frequency fluctuations of wall shear stress was accompanied by a reduction in the rms value above 10 Hz (the voltmeter cutoff frequency). The rms ratio (with to without injection) was 0.52, 0.60, and 0.63 in order of decreasing concentrations. When the normalized spectral densities are calculated, as shown in Fig. 9, one obtains a representation of the actual turbulent fluctuations present as explained in Appendix B. In this case, a higher percentage of turbulent energy is contained in the frequency range below approximately 30 Hz when the polymer is injected. This is accompanied by a severe attenuation of the higher frequencies up to the signal-to-noise ratio limit of the tape recorder. It should be mentioned that the confidence level in the low-frequency range is rather poor since the tape loops had a running time of only 2.4 sec. The low-frequency scatter in the spectral density output from the analyzer was smoothed by eye.

Gadd[42] in his dye studies on polyox jets, observed that the suppression of small-scale turbulence did not seem to occur below concentrations of 10 ppm, while 30 ppm seemed to be very effective. The much lower "effective concentrations" in this report must be interpreted with caution since outward diffusion as the boundary layer moved downstream definitely reduced the spectral density reduction. Thus, the forward station was affected by a much higher sublayer concentration than was calculated. It is unfortunate that there is no method presently for obtaining the true local concentration

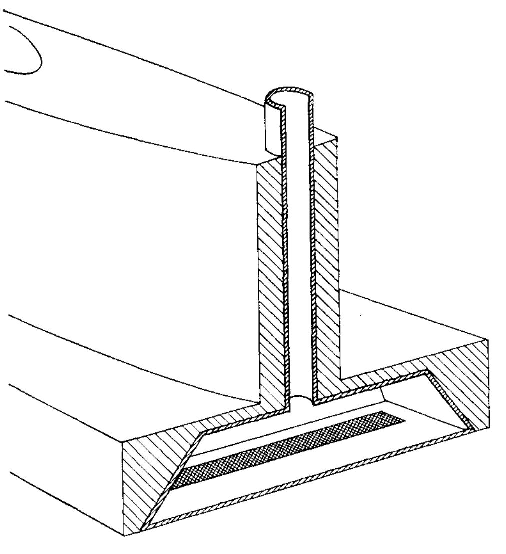

Fig. 5 Section AA′ injection tank.

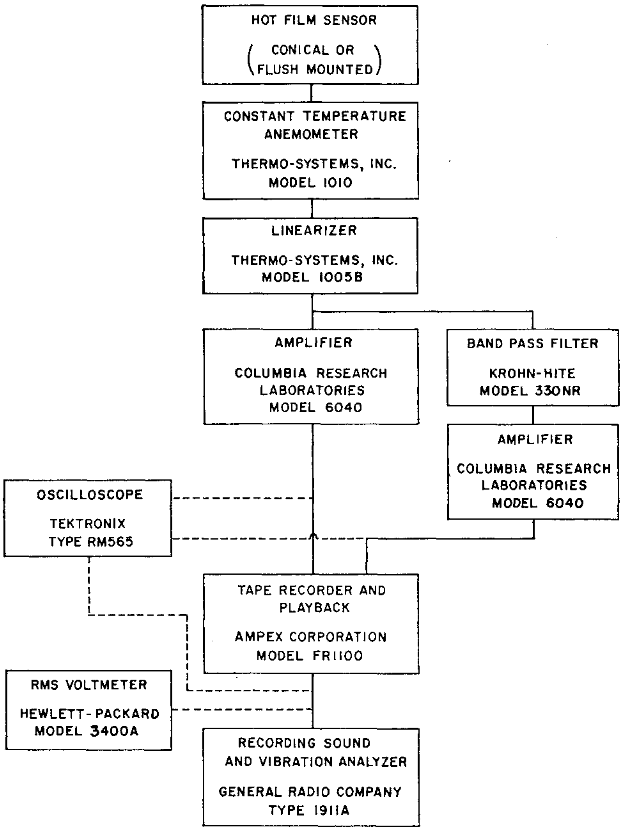

Fig. 6 Instrumentation.

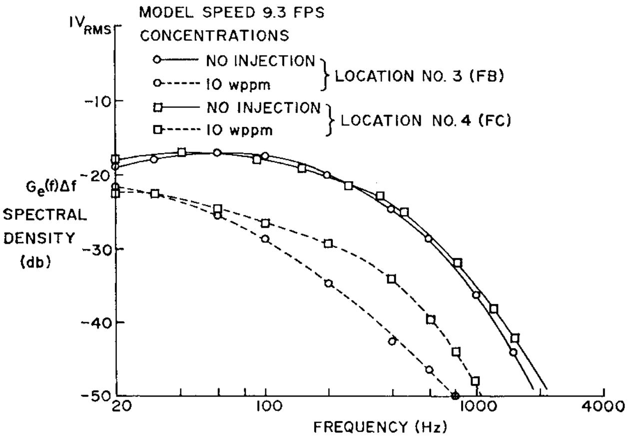

Fig. 7 Spectral density (db) vs frequency (Hz) with and without injection of 0.5% polyox coagulant in aqueous formaldehyde solution (bandwidth equals $\frac{1}{10}$ octave).

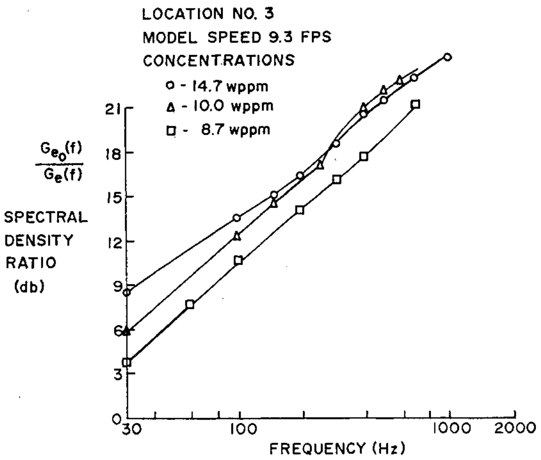

Fig. 8 Spectral density ratio (db) vs frequency (Hz) for 0.5% polyox coagulant in aqueous formaldehyde solution.

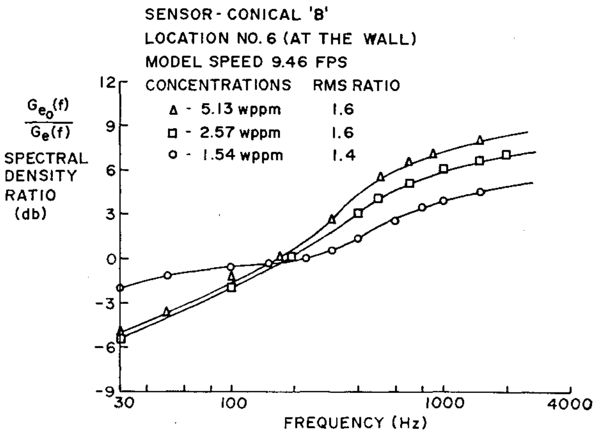

Fig. 10 Spectral density ratio (db) vs frequency (Hz) for 0.128% polyox coagulant in aqueous formaldehyde solution.

in the sublayer. Sampling probes are just too large to sample in the viscous sublayer. This area needs further study.

The optimum average concentration at the forward station for maximum rms reduction was 8.6 ppm. The sublayer concentration would probably be at least three times the mean concentration.

Figure 10 illustrates the influence of very small concentrations of polymer in the wall turbulent region (at about 0.01 in. from the wall). In this case, the conical sensor is considered to measure the longitudinal velocity fluctuations along the plate. In contrast with the flush-mounted measurements, however, the rms values rose considerably with injection of the additive. This increase is caused by the increased spectral density below 200 Hz at a velocity of 9.5 fps. Since the spectral density again decreases above 200 Hz, it appears that the turbulence energy is concentrated in the large eddies which do not dissipate so rapidly as the small eddies. This result is consistent with the results of Vogel and Patterson,[18] Virk,[14] and Patterson and Zakin.[7,8]

Discussion

Drag is associated with the dissipation of energy. The conversion of mean flow energy to turbulence energy and thence to thermal energy is commonly called the cascade process. In a boundary layer, the production and dissipation of turbulence energy takes place primarily in the region just outside the viscous sublayer where a high product of the Reynolds stresses and mean velocity gradients is found. According to Townsend,[43] in a turbulent energy balance, the turbulent energy production term is represented by $\rho\overline{uv}(d\bar{U}/dy)$.

It should be noted that production of turbulence is an inertial phenomenon involving momentum transfer on a macroscopic scale. It is not directly governed by the viscosity of the fluid, which is representative of momentum transfer on a molecular scale. Boundary-layer shearing motions produce a spectrum of turbulent eddy scales, but viscous dissipation involves only the smallest eddies decaying into thermal energy. There is an approximate balance between turbulent energy production and dissipation at any location in a boundary layer.[43,44]

In this investigation the observed shift to a predominately large eddy structure suggests the nature of drag reduction when polymer additives are introduced into a turbulent boundary layer. If the production of dissipative eddies is decreased by the addition of long chain molecules, the resulting larger eddy structure will both remove and replace nearly equal amounts of freestream energy, i.e., there will be a smaller energy deficit which must be supplied by the freestream to maintain the turbulent velocity profile. Although the net momentum deficit caused by dissipation is decreased by polymer injection, the total momentum transport is probably not decreased as suggested by Walsh,[24] since the rms level of the fluctuation increased in the inner regions of the boundary layer.

Investigations of the viscous sublayer are most helpful in gaining a physical picture of the cause of the apparent shift in turbulent scales with the addition of polymer additives. The concept of vortex stretching in the high shear regions of the boundary layer has been discussed by Theodorsen,[45] Sternberg,[46] Stuart,[47] Tu and Wilmarth,[48] and Bakewell.[49]

Physically, it seems possible that turbulence production results from the alternate stretching, colliding, and collapsing of the spaghetti-like vortices present in a turbulent boundary layer, e.g., fluid mechanics films by Kline, Rouse, Taylor, and Shapiro. The greatest stretching occurs when the vortex is in the steepest part of the mean velocity gradient. If the vortex core is attached to a small roughness element at the wall and extends outward into the flow, it is obvious that considerable stretching will occur. As this turbulent eddy is stretched, the velocity near the core increases (as in a tornado) as suggested by Helmholtz' vorticity laws, thus increasing both the intensity and frequency relative to the sensing element. This stretching continues until the eddy interacts with another eddy, slowing down the combination but possibly spinning off some very small eddies which rapidly dissipate. The presence of polymer additives in these turbulent eddies is felt to reduce the tendency of the vortex to increase in intensity and decrease in size as it is stretched.

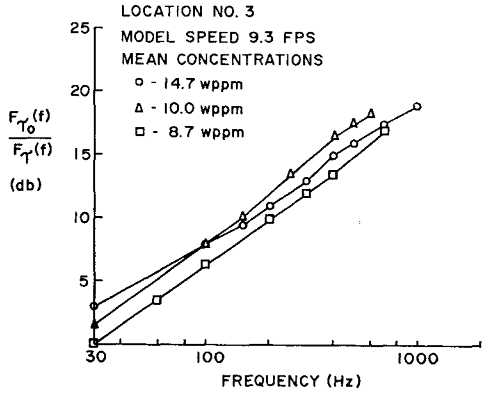

Fig. 9 Normalized spectral density ratio (db) vs frequency (Hz) for 0.5% polyox coagulant in aqueous formaldehyde solution.

There are at least two ways in which an additive could inhibit vortex stretching in the boundary layer. The first involves an effect on the inertial character of turbulence.[23] It is possible that the polar moment of inertia perpendicular to the longitudinal axis of the molecule (assuming that the coils tend to straighten as they are spun by the eddy) provides sufficient inertial resistance to rotational acceleration to decrease the underpressure in the vortex core and reduce the production of small scale turbulence. This resistance to rotational acceleration could explain the drag reduction found in fiber suspensions[50] where no viscoelasticity is present.

A second influence on vortex stretching involves the effect of viscoelastic chain entanglement. A high polymer concentration in the sublayer may increase resistance to vortex stretching as suggested by Metzner and Seyer.[28] For a given shear, elastic chain entanglement would have the effect of increasing the thickness of the viscous sublayer since less turbulent energy would be convected to the wall when the intensity of the vorticity is lowered through the resistance to stretching.

Actual flow conditions are probably a combination of the inertial and viscoelastic effects, with one or the other dominating, depending on the local polymer concentration, entanglement of the polymer chains, and stretched out molecular length. These two possibilities could explain the different mechanisms observed by Gadd[51] when comparing threshold concentrations of different polymers in vortex street experiments.

Conclusions

The following conclusions and recommendations are made on the basis of the results of this preliminary investigation:

1) The primary influence of polymer additives on a turbulent boundary layer is to decrease the production of small scale turbulence. This decreases the dissipation of turbulence energy and therefore decreases skin friction drag.

2) The injection of a polymer solution into a turbulent boundary layer reduced the spectral density of the turbulent fluctuations at the wall across the entire frequency spectrum. The mean wall shear stress and the root mean square of the fluctuations in wall shear stress were also reduced.

3) Polymer injection *increased* the root mean square of the *velocity* fluctuations in the wall turbulent region The spectral density of these fluctuations increased below 200 Hz but decreased above 200 Hz at this Reynolds number indicating that the large eddies tend to persist in the presence of the polymer and do not degenerate into dissipative eddies so rapidly.

4) A decrease in the reduction of turbulence scales was noted in moving aft on the model. This probably indicates an accompanying decrease in drag reduction. Thus, in any application of this phenomenon involving injection, it would be better to inject at several points along the boundary rather than at just one.

5) A calibration of the hot films and calculation of the wall shearing stress using the data from the flush-mounted hot films are necessary. This would give a true picture of the time dependent velocity structure in the sublayer.

6) Much more work needs to be done in the investigation of the artificial turbulent structure caused by the trip and on the effects of polyox diffusion across the boundary layer. This work is necessary to pin-point its effect on the generation and dissipation of turbulence.

Appendix A: Interpretation of Anemometer Signals

The heat-transfer equation which is assumed to govern the relationship between the input to the hot-film sensor (U) and the bridge voltage output of the anemometer (E_b) is a generalized King's Formula[52,53]

$$KE_b^2 = (A + BU^n)\Delta t \tag{A1}$$

For constant temperature operation, A and B can absorb the constants K and Δt.

Differentiating Eq. (A1) we obtain

$$2E_b dE_b = BnU^{n-1}dU + \text{higher-order terms}$$

which combined with Eq. (A1) gives

$$2E_b dE_b/(E_b^2 - A) = n(dU/U) + \text{higher-order terms}$$

Assuming $E_b^2 \gg A$, the operating point is in a region where the local curvature of the E_b^2 vs U curve is negligible, the higher-order terms are generally neglected giving

$$2E_b dE_b/(E_b^2 - A) = n(dU/U) \tag{A2}$$

We now let

$$E_b(t) = \bar{E}_b + dE_b(t) = \bar{E}_b + e$$

$$U(t) = \bar{U} + dU(t) = \bar{U} + u$$

$$\tau_w(t) = \bar{\tau}_w + d\tau_w(t) = \bar{\tau}_w + \tau_w'$$

In other words, each variable is considered to be made up of a time-averaged (d.c.) plus a fluctuating (a.c.) component.

Noting that $e_{\text{rms}} = (\overline{e^2})^{1/2}$, etc., we substitute the time-averaged fluctuating quantities into Eq. (2) to obtain

$$[2\bar{E}_b/(\bar{E}_b^2 - A)]e_{\text{rms}} = n(u_{\text{rms}}/\bar{U}) \tag{A3}$$

Thus there is no simple relationship between the velocity fluctuations and the bridge voltage fluctuations. Either Eq. (A3) must be solved for each data point or a linearizer may be used.

If a linearizer is used, the following equations simplify the analysis. Let $E_L = \bar{E}_L + e_L$ = output of linearizer

$$E_L = (E_b^2 - A)^m = (BU^n)^m \tag{A4}$$

If the linearizer circuit is set for $m = (1/n)$ then

$$E_L = B^{1/n}U = KU \tag{A5}$$

and

$$e_L = Ku \tag{A6}$$

The turbulence intensity then becomes easy to measure as

$$T.I. \equiv u_{\text{rms}}/\bar{U} = e_{L\text{rms}}/\bar{E}_L \tag{A7}$$

This linear relationship between voltage and velocity fluctuations is difficult to obtain, however, as A, B, and n all vary among various types of hot-film sensors.[7,13,28,52,53]

Appendix B: Interpretation of Spectral Density Measurements

The power spectral density function $G_e(f)$, is defined in Ref. 54 as follows:

$$G_e(f) = \lim_{\Delta f \to 0} \frac{e_{\text{rms}}^2(f, f + \Delta f)}{\Delta f}$$

or

$$e_{\text{rms}}^2 = \int_0^\infty G_e(f)df \tag{B1}$$

However, most spectral density analyzers measure $\{G_e(f)\Delta f\}$ vs f for constant or constant percentage bandwidths, Δf.

By taking spectral density ratios with and without injection of polymer additives, the bandwidth drops out (except for the errors involved with finite bandwidths) and one can present a plot of spectral density ratio in decibels vs frequency. Recording analyzers usually plot the output in db since 20db represents either an order of magnitude change in voltage level or two orders of magnitude change in energy or power spectra level, i.e.,

$$n(\text{db}) = 20 \log_{10}(e_{\text{rms}}/e_{\text{rms}0}) = 10 \log_{10}(e_{\text{rms}}^2/e_{\text{rms}0}^2)$$

The ratio is easy to compute since it is the numberical difference in db between the two levels being compared.

To interpret the spectral density distribution as representative of the turbulent eddy scale distribution is very difficult, however. First of all, if the mean velocity varies during the experiment, the frequencies should be converted to one-dimensional wave numbers k_1, using[39]

$$k_1 = 2\pi f/U_c \tag{B2}$$

where U_c is the velocity at which the eddy is convected past the sensor. This was not done for the results of this experiment, since the mean velocity was held constant and U_c was not known for the flush-mounted sensors.

We also note from Eq. (A3) that

$$G_u(f) = [2\bar{E}_b\bar{U}/n(\bar{E}_b^2 - A)]^2 G_e(f) \tag{B3}$$

so that the absolute level of the spectral density measurements of the velocity or wall shear stress fluctuations is difficult to calculate without a complete knowledge of the variables in the coefficient. The coefficient does not drop out when ratios are taken since both $\bar{E}_b$ and $\bar{U}$ changed when the polymer was injected.

This problem is partly solved by defining a normalized spectral density $F_e(f)$

$$F_e(f) = G_e(f)/e_{\text{rms}}^2 = G_u(f)/u_{\text{rms}}^2 = F_u(f), \text{ sec} \tag{B4}$$

so that

$$\int_0^\infty F_e(f)df = 1 = \int_0^\infty F_u(f)df$$

and linearization is not necessary if $E_b^2 \gg A$.

The normalized density is a measure of the relative amount of turbulence energy at any one frequency or wave number. Therefore, if one end of the spectrum is depressed, some other segment must be increased. To determine the influence of the polymer on the absolute spectral density, at any given wave number, calculation of $G_u(f)$, would be greatly enhanced by proper linearization of the signals.

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